UNIVERSAL GRÖBNER BASES AND CARTWRIGHT–STURMFELS IDEALS

ALDO CONCA

We will discuss a family of multigraded ideals, that we name after Cartwright and Sturmfels, defined in terms of properties of the multigraded generic initial ideals. Indeed, by definition, a multigraded ideal I is a Cartwright–Sturmfels ideal if it has a radical multigraded generic initial ideal. Our main technical result asserts that the family of Cartwright–Sturmfels ideals is closed under several natural operations including multigraded linear sections and multigraded eliminations. Connection to universal Gröbner bases for determinantal ideals, algebras associated to graphs, subspaces configurations and multiview varieties will be discussed. We will also present a "rigidity" conjecture suggested by a theorem of Brion.

This is a report on a joint work with Emanuela De Negri and Elisa Gorla that appeared in a series of four papers we wrote together:

[1] *Universal Gröbner bases for maximal minors.*
Int. Math. Res. Not. (2015), no. 11, 3245–3262.

[2] *Universal Gröbner bases and Cartwright-Sturmfels ideals*
preprint 2016, arXiv:1608.08942

[3] *Multigraded generic initial ideals of determinantal ideals*
preprint 2016, arXiv:1608.08944

[4] *Cartwright-Sturmfels ideals associated to graphs and linear spaces*
preprint 2017, soon on arxiv.

FINITE GENERATION OF EXTENSIONS OF ASSOCIATED GRADED RINGS ALONG A VALUATION

STEVEN DALE CUTKOSKY

Suppose that K is a field. Associated to a valuation ν of K is a value group Φ_ν and a valuation ring V_ν with maximal ideal m_ν. Let R be a local domain with quotient field K which is dominated by ν. We have an associated semigroup $S^R(\nu) = \{\nu(f) \mid f \in R\}$, as well as the associated graded ring of R along ν

$$\mathrm{gr}_\nu(R) = \bigoplus_{\gamma\in\Phi_\nu} \mathscr{P}_\gamma(R)/\mathscr{P}_\gamma^+(R) = \bigoplus_{\gamma\in S^R(\nu)} \mathscr{P}_\gamma(R)/\mathscr{P}_\gamma^+(R)$$

which is defined by Teissier in [3]. Here

$$\mathscr{P}_\gamma(R) = \{f \in R \mid \nu(f) \geq \gamma\} \text{ and } \mathscr{P}_\gamma^+(R) = \{f \in R \mid \nu(f) > \gamma\}.$$

This ring plays an important role in local uniformization of singularities ([3] and [4]). The ring $\mathrm{gr}_\nu(R)$ is a domain, but it is often not Noetherian, even when R is.

Suppose that K^* is a finite separable extension of K and ν^* is an extension of ν to K^*.

In this talk we consider the question of when the associated graded ring along a valuation, $\mathrm{gr}_{\nu^*}(S)$, is a finite $\mathrm{gr}_{\nu^*}(R)$-module, where S is the normal local ring of K which is the localization of the integral closure of R in K^* at the center of ν^*.

We begin by discussing some examples and results allowing us to refine the conditions under which finite generation can hold. We must impose the condition that the extension of valuations is *defectless* and perform a birational extension of R along the valuation to obtain finite generation (replacing S with the local ring of K^* determined by the valuation). With these assumptions, we show that finite generation holds, when R is a two dimensional excellent local ring.

Our main result (in [2]) is to show that for an arbitrary valuation in an algebraic function field over an arbitrary field of characteristic zero, after a birational extension along the valuation, we always have finite generation (all finite extensions of valued fields are defectless in characterisitic zero). This generalizes an earlier result, [1], showing that finite generation holds (after a birational extension) with the additional assumptions that ν has rank 1 and has an algebraically closed residue field. We discuss some of the difficulties involved in extending this result to arbitrary rank. As an ingredient in the proof, we obtain general results for unramified extensions of excellent local rings.

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MYSTERIES OF FREE RESOLUTIONS OVER COMPLETE INTERSECTIONS

DAVID EISENBUD

Despite recent progress in understanding the free resolution of a finitely generated module M over a complete intersections R (work of mine with Irena Peeva, Frank Schreyer and others) there remain many open questions. I'll review the new structure theorems, and survey some of these questions.

ASYMPTOTIC SYZYGIES

DANIEL ERMAN

I'll discuss some new computational and theoretical results related to asymptotic syzygies. For instance, I'll explain how we can use random monomial ideals to provide an example of a family of ideals where the Betti numbers converge to a normal distribution. This is joint work with Jay Yang.

ON A STRATIFICATION OF COHEN–MACAULAY RINGS

SHIRO GOTO

My lecture is based on the work jointly with S. Kumashiro [3] and purposes to give a survey on generalized Gorenstein local rings. The generalization of Gorenstein local rings in my sense dates back to the paper of V. Barucci and R. Fröberg [1] in 1997, where they introduced the notion of an almost Gorenstein local ring (AGL for short) to one dimensional analytically unramified local rings. In 2013, the author, N. Matsuoka, and T. T. Phuong [5] gave a new definition of an AGL ring for arbitrary but still one-dimensional Cohen-Macaulay local rings. This research has been succeeded by two works of T. D. M. Chau, the author, S. Kumashiro, N. Matsuoka [2], and the author, R. Takahashi, and N. Taniguchi [11] in 2017 and 2015, respectively. In the former work, one can find the notion of a 2-almost Gorenstein local ring (2-AGL ring for short) of dimension one, which is a natural generalization of AGL rings. Using Sally modules of canonical ideals, the authors show that 2-AGL rings behave well as if they were twins of AGL rings. The latter research started in a different direction. They have extended the notion of an AGL ring to higher dimensional Cohen-Macaulay local/graded rings, using the notion of Ulrich modules with respect to the maximal ideal. Researches on AGL rings are in progress, exploring, e.g., the problem of when the Rees algebras of ideals/modules are almost Gorenstein graded rings ([4, 6, 7, 8, 9, 10]). The present purpose is to report one more notion, which I would like to call a *generalized Gorenstein local ring* (GGL for short), and which might be a more reasonable stratification of Cohen-Macaulay rings, including the whole class of AGL rings and a part (not the whole class) of 2-AGL rings.

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ORDINARY AND SYMBOLIC POWERS AND THE GOLOD PROPERTY

JÜRGEN HERZOG

Let R be a standard graded K-algebra with graded maximal ideal $\mathfrak{m}$. The formal power series $P_R(t) = \sum_{i\geq 0} \dim_K \operatorname{Tor}_i(R/\mathfrak{m}, R/\mathfrak{m})t^i$ is called the Poincaré series of R. In general, $P_R(t)$ is not a rational series. However, Serre showed that $P_R(t)$ is coefficientwise bounded above by the rational series

$$\frac{(1+t)^n}{1 - t\sum_{i\geq 1} \dim_K H_i(\mathbf{x}; R)t^i},$$

where $\mathbf{x} = x_1, \ldots, x_n$ is a minimal system of generators of $\mathfrak{m}$ and where $H_i(\mathbf{x}; R)$ denotes the ith Koszul homology of the sequence $\mathbf{x}$.

The ring R is called Golod, if $P_R(t)$ coincides with this upper bound given by Serre. Obviously the residue field of a Golod ring has a rational Poincaré series.

Suppose $R = S/I^k$, where $S = K[x_1, \ldots, x_n]$ is the polynomial ring over a field K and I is a graded ideal. In this lecture we report on a joint result with Craig Huneke, in which we show that if the characteristic of K is zero, then R is Golod for all $k \geq 2$. The same holds true for the symbolic and saturated powers of I. However the corresponding result in positive characteristic is still open. Only recently, in a joint paper with Maleki, we showed that S/I^k is Golod in all characteristics if I is a monomial ideal. A local version of the theorem is also missing.

The method to derive the above mentioned results is based on an explicit description of the Koszul cycles representing the homology classes of $H_i(\mathbf{x}; R)$. This description is given in terms of the data provided by the minimal free S-resolution of $R = S/I$.

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GEIGLE-LENZING COMPLETE INTERSECTIONS AND TATE RESOLUTIONS

OSAMU IYAMA

Let $d \geq -1$ and $n \geq 0$ be integers. For positive integers $p_1, \dots, p_n$, we define a *Geigle-Lenzing complete intersection* [3] as

$$S := k[T_0, \dots, T_d, X_1, \dots, X_n]/(X_i^{p_i} - \ell_i(T_0, \dots, T_d) \mid 1 \leq i \leq n),$$

where $\ell_1, \dots, \ell_n$ are linear forms on $T_0, \dots, T_d$ in a general position. The ring S has Krull dimension $d+1$, and is canonically graded by an abelian group

$$\mathbb{L} := \langle \vec{c}, \vec{x}_1, \dots, \vec{x}_n \rangle / \langle p_i \vec{x}_i - \vec{c} \mid 1 \leq i \leq n \rangle$$

of rank 1 by $\deg X_i = \vec{x}_i$ for $1 \leq i \leq n$ and $\deg T_j = \vec{c}$ for $0 \leq j \leq d$.

In the case $d = 1$, the representation theory of S was initiated by Geigle-Lenzing [2] as a large extension of Auslander's results on simple surface singularities [1].

I will discuss the category $\mathsf{CM}^{\mathbb{L}} S$ of $\mathbb{L}$-graded maximal Cohen-Macaulay S-modules for arbitrary d. We need the *a-invariant* and the *dominant element* given by

$$\vec{\omega} = (n-d-1)\vec{c} - \sum_{i=1}^{n} \vec{x}_i \ \text{ and } \ \vec{\delta} = d\vec{c} + 2\vec{\omega} \ \text{ respectively.}$$

A main result in [3] is the following, where $\mathsf{mod}^{[0,\vec{\delta}]} S$ is the category of finitely generated $\mathbb{L}$-graded S-modules X which are concentrated in degrees in the interval $[0, \vec{\delta}]$.

Theorem [3] *There is an equivalence of triangulated categories*

$$\underline{\mathsf{CM}}^{\mathbb{L}} S \simeq \mathsf{D}^{\mathsf{b}}(\mathsf{mod}^{[0,\vec{\delta}]} S).$$

The abelian category $\mathsf{mod}^{[0,\vec{\delta}]} S$ is equivalent to the category of finitely generated modules over a finite dimensional k-algebra A, an analogue of Ringel's canonical algebra.

In this talk, I will give an explicit formula of their global dimension. This provides us with a family of S which are d-Cohen-Macaulay finite. A crucial step is to give an $\mathbb{L}$-graded version of Tate's DG algebra resolution for complete intersection rings [4].

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ATOM-MOLECULE CORRESPONDENCE AND CLASSIFICATION OF SUBCATEGORIES FOR LOCALLY NOETHERIAN SCHEMES

RYO KANDA

For a commutative noetherian ring, prime ideals bijectively correspond to isoclasses of indecomposable injectives, which is a result due to Matlis. For a noncommutative noetherian ring, isoclasses of indecomposable injectives and prime (two-sided) ideals do not necessarily correspond bijectively, but there still exist canonical maps between them, which were introduced by Gabriel [Gab62]. In this talk, we establish Gabriel's maps in a more general setting, and show that the bijectivity of those maps still holds for the category $\operatorname{QCoh} X$ of quasi-coherent sheaves on an arbitrary locally noetherian scheme X.

We will work on a Grothendieck category, which is a generalization of both the category of modules over a noncommutative ring and the category of quasi-coherent sheaves on a scheme. We introduce the notions of *atoms* and *molecules* in the Grothendieck category. They are reformulations of indecomposable injectives and prime ideals. The relationship between atoms and molecules is more direct than that between injectives and primes, and this reformulation reveals some strong connections between these two notions even in the case of the module category of a noncommutative noetherian ring.

For a locally noetherian scheme, we show that atoms and molecules bijectively correspond to each other. Moreover, they also correspond to certain classes of subcategories:

Theorem 1. *Let $X = (|X|, \mathscr{O}_X)$ be a locally noetherian scheme. Then the following collections bijectively correspond to each other:*

- *points of the underlying space $|X|$,*
- *isomorphism classes of indecomposable injective objects in* $\operatorname{QCoh} X$*,*
- *atoms in* $\operatorname{QCoh} X$*,*
- *molecules in* $\operatorname{QCoh} X$*,*
- *prime localizing subcategories of* $\operatorname{QCoh} X$*,*
- *prime closed subcategories of* $\operatorname{QCoh} X$*, and*
- *prime quasi-coherent subsheaves of the structure sheaf $\mathscr{O}_X$.*

The proof uses the classification of subcategories given in [Kan15].

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ON THE RELATION BETWEEN REPRESENTATION THEORY AND HIBI RINGS

SANGJIB KIM

I will give a survey on the relation between representation theory and Hibi rings. Recently, Hibi rings and distributive lattices have been studied extensively in the context of finite dimensional representations of reductive complex algebraic groups. With some explicit examples, I will illustrate how Hibi rings and combinatorial objects associated with them can be used to describe representation theoretic information of certain invariant rings. This is based on a series of joint works with Roger Howe and Soo Teck Lee.

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INFINITELY GENERATED SYMBOLIC REES RINGS OF SPACE MONOMIAL CURVES HAVING NEGATIVE CURVES

KAZUHIKO KURANO

This is a joint work [7] with Koji Nishida (Chiba University).

Let K be a field and $S = K[x,y,z]$ be a polynomial ring with three variables. Let $\mathfrak{p}_K(a,b,c)$ be the ideal of S which defines the space monomial curve (t^a,t^b,t^c) for $\underline{\text{pairwise coprime integers } a, b, c}_{\text{(i)}}$. My interest is whether the symbolic Rees ring

$$\mathscr{R}_s(\mathfrak{p}_K(a,b,c)) = \oplus_{i\geq 0}\mathfrak{p}_K(a,b,c)^{(n)}T^n \subset S[T]$$

is finitely generated or not. Remember that $\mathfrak{p}_K(a,b,c)$ is an ideal of height 2, and generated by at most three elements (Herzog [5]).

The symbolic Rees rings of space monomial primes are deeply studied by many authors. Huneke [6] gave a criterion for finite generation of $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$.

Cutkosky [1] found the geometric meaning of the symbolic Rees rings of space monomial primes. Let $\mathbb{P}_K(a,b,c)$ be the weighted projective surface with degree a, b, c. Let $\pi : X_K(a,b,c) \longrightarrow \mathbb{P}_K(a,b,c)$ be the blow-up at the point corresponding to $\mathfrak{p}_K(a,b,c)$. Then, the Cox ring of $X_K(a,b,c)$ is equal to the extended symbolic Rees ring of the space monomial prime $\mathfrak{p}_K(a,b,c)$. Therefore, the symbolic Rees ring of the space monomial prime $\mathfrak{p}_K(a,b,c)$ is finitely generated if and only if the Cox ring of $X_K(a,b,c)$ is finitely generated, that is, $X_K(a,b,c)$ is a Mori dream space. A curve C on $X_K(a,b,c)$ is called the negative curve if $C^2 < 0$ and C is different from the exceptional curve E. $\underline{\text{Now suppose } \sqrt{abc} \notin \mathbb{Q} \text{ for simplicity}}_{\text{(ii)}}$. Cutkosky [1] proved that $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated if and only if the following two conditions are satisfied:

(1) There exists a negative curve C.
(2) There exists a curve D on $X_K(a,b,c)$ such that $C \cap D = \emptyset$.

Two equations defining $\pi(C)$ and $\pi(D)$ as above satisfy Huneke's criterion. Let f be an irreducible polynomial which defines $\pi(F)$ for a curve F on $X_K(a,b,c)$. Let d be the degree of f. Let r be the integer satisfying $f \in \mathfrak{p}_K(a,b,c)^{(r)} \setminus \mathfrak{p}_K(a,b,c)^{(r+1)}$. Then, F is a negative curve if and only if $d/r < \sqrt{abc}$.

If $\mathfrak{p}_K(a,b,c)$ is generated by two elements, then the symbolic power coincides with the ordinary power. So, $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated in this case.

In the rest, suppose that $\underline{\mathfrak{p}_K(a,b,c) \text{ is minimally generated by three polynomials}}_{\text{(iii)}}$.

As long as I know, there is no example that $X_K(a,b,c)$ does not have a negative curve. If $X_K(a,b,c)$ did not have a negative curve for some field K, then Nagata conjecture for abc points would be true [2], that is, if a curve of degree d is passing through general abc points in $\mathbb{P}^2_{\mathbb{C}}$ with multiplicity at least r at each abc points, then $d > \sqrt{abc}r$.

In the case where the characteristic of K is positive, Cutkosky [1] proved that the symbolic Rees ring $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated if and only if there exists a negative curve on $X_K(a,b,c)$. There is no example that $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is infinitely generated when the characteristic of K is positive.

In th rest, we assume that $\underline{\text{the characteristic of } K \text{ is zero}}_{\text{(iv)}}$. There are many examples of finitely generated $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$. However, the first example of infinitely generated $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ was given by Goto-Nishida-Watanabe [4], for example, $\mathscr{R}_s(\mathfrak{p}_K(25,29,72))$. Recently Gonzáles-Karu [3] found some sufficient condition for infinite generation of the symbolic Rees ring $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$. In these examples, the element of $\mathfrak{p}_K(a,b,c)$ with minimal degree is the equation of the image of the negative curve.

In the rest, we assume that the element in $\mathfrak{p}_K(a,b,c)$ with minimal degree, say $\underline{z^u - x^{s_3}y^{t_3}}$, $\underline{\text{is the equation of the image of the negative curve}}_{\text{(v)}}$.

Remark that, under these assumptions, the symbolic Rees ring $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated if and only if there exists $\eta' \in [\mathfrak{p}_K(a,b,c)^{eab}]_{eu}$ such that $z^u - x^{s_3}y^{t_3}$, η' satisfy Huneke's criterion **for some sufficiently divisible** e.

Our following main theorem says that we have only to check the case $e = 1$.

Theorem *Under these assumptions (i), (ii), (iii), (iv), (v) as above, the symbolic Rees ring $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated if and only if there exists $\eta \in [\mathfrak{p}_K(a,b,c)^{ab}]_u$ such that $z^u - x^{s_3}y^{t_3}$, η satisfy Huneke's criterion.*

This result says that the symbolic Rees ring $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ is finitely generated if and only if there exists a curve D with $C \cap D = \emptyset$ such that $D.E = u$.

By this result, under these assumptions, we can decide whether the symbolic Rees ring is Notherian using computers.

For example, $(17,503,169)$ does not satisfy the sufficient condition for infinite generation due to Gonzáles-Karu [3]. However, we know that the symbolic Rees ring is infinitely generated by the above theorem and calculation using computers.

In order to prove the above theorem, we need Cutkosky's method [1] in characteristic positive, Fujita's vanishing theorem, and mod p reduction introduced in Goto-Nishida-Watanabe [4]. The key point is that the negative curve is isomorphic to $\mathbb{P}^1_K$ in this case.

Furthermore, under these assumptions, we give a very simple necessary and sufficient condition for finite generation of symbolic Rees rings $\mathscr{R}_s(\mathfrak{p}_K(a,b,c))$ when $u \leq 6$.

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DEGREE BOUNDS FOR LOCAL COHOMOLOGY

CLAUDIA POLINI

In this talk I will show how to estimate degrees of generators of local cohomology modules. I will also survey several applications to Rees algebras, to hyperplane sections, and to symbolic powers. This is joint work with Andy Kustin and Bernd Ulrich.

GRADED COMPONENTS OF LOCAL COHOMOLOGY MODULES

TONY J. PUTHENPURAKAL

Standard Assumption: From henceforth A will denote a regular ring containing a field of characteristic zero. Let $R = A[X_1, \ldots, X_m]$ be standard graded with $\deg A = 0$ and $\deg X_i = 1$ for all i. We also assume $m \geq 1$. Let I be a homogeneous ideal in R. Set $M = H^i_I(R)$. It is well-known that M is a graded R-module. Set $M = \bigoplus_{n \in \mathbb{Z}} M_n$.

I: *(Vanishing:)* The first result we prove is that vanishing of almost all graded components of M implies vanishing of M. More precisely we show

Theorem I: *If $M_n = 0$ for all $|n| \gg 0$ then $M = 0$.*

II (*Tameness:*) In view of Theorem I, it follows that if $M = H^i_I(R) = \bigoplus_{n \in \mathbb{Z}} M_n$ is *non-zero* then either $M_n \neq 0$ for infinitely many $n \ll 0$, OR, $M_n \neq 0$ for infinitely many $n \gg 0$. We show that M is *tame*. More precisely

Theorem II

(a) *The following assertions are equivalent:*
  (i) *$M_n \neq 0$ for infinitely many $n \ll 0$.*
  (ii) *$M_n \neq 0$ for all $n \leq -m$.*
(b) *The following assertions are equivalent:*
  (i) *$M_n \neq 0$ for infinitely many $n \gg 0$.*
  (ii) *$M_n \neq 0$ for all $n \geq 0$.*

III (*Rigidity:*) Surprisingly non-vanishing of a single graded component of $M = H^i_I(R)$ is very strong. We prove the following rigidity result:

Theorem III

(a) *The following assertions are equivalent:*
  (i) *$M_r \neq 0$ for some $r \leq -m$.*
  (ii) *$M_n \neq 0$ for all $n \leq -m$.*
(b) *The following assertions are equivalent:*
  (i) *$M_s \neq 0$ for some $s \geq 0$.*
  (ii) *$M_n \neq 0$ for all $n \geq 0$.*
(c) *(When $m \geq 2$.) The following assertions are equivalent:*
  (i) *$M_t \neq 0$ for some t with $-m < t < 0$.*
  (ii) *$M_n \neq 0$ for all $n \in \mathbb{Z}$.*

IV *(Infinite generation:)* We give a sufficient condition for infinite generation of a component of graded local cohomology module over R.

Theorem IV *Further assume A is a domain. Assume $I \cap A \neq 0$. If $H^i_I(R)_c \neq 0$ then $H^i_I(R)_c$ is NOT finitely generated as an A-module.*

V (*Bass numbers:*) The j^{th} Bass number of an A-module E with respect to a prime ideal P is defined as $\mu_j(P,E) = \dim_{k(P)} \ Ext^j_{A_P}(k(P), E_P)$ where $k(P)$ is the residue field of A_P. We note that if E is finitely generated as an A-module then $\mu_j(P,E)$ is a finite number (possibly zero) for all $j \geq 0$. In view of Theorem IV it is not clear whether $\mu_j(P, H^i_I(R)_n)$ is a finite number. Surprisingly we have the following dichotomy:

Theorem V *Let P be a prime ideal in A. Fix $j \geq 0$. EXACTLY one of the following hold:*

(i) *$\mu_j(P,M_n)$ is infinite for all $n \in \mathbb{Z}$.*
(ii) *$\mu_j(P,M_n)$ is finite for all $n \in \mathbb{Z}$.*

VI (*Growth of Bass numbers*). Fix $j \geq 0$. Let P be a prime ideal in A such that $\mu_j(P, H^i_I(R)_n)$ is finite for all $n \in \mathbb{Z}$. We may ask about the growth of the function $n \mapsto \mu_j(P, H^i_I(R)_n)$ as $n \to -\infty$ and when $n \to +\infty$. We prove

Theorem VI *Let P be a prime ideal in A. Let $j \geq 0$. Suppose $\mu_j(P,M_n)$ is finite for all $n \in \mathbb{Z}$. Then there exists polynomials $f^{j,P}_M(Z), g^{j,P}_M(Z) \in \mathbb{Q}[Z]$ of degree $\leq m-1$ such that*

$$f^{j,P}_M(n) = \mu_j(P,M_n) \text{ for all } n \ll 0 \quad \text{AND} \quad g^{j,P}_M(n) = \mu_j(P,M_n) \text{ for all } n \gg 0.$$

VII (*Associate primes:*) For associate primes of graded components of local cohomology modules we prove:

Theorem VII *Further assume that either A is local or a smooth affine algebra over a field K of characteristic zero. Let $M = H^i_I(R) = \bigoplus_{n\in\mathbb{Z}} M_n$. Then*

(1) *$\bigcup_{n\in\mathbb{Z}} Ass_A M_n$ is a finite set.*
(2) *$Ass_A M_n = Ass_A M_m$ for all $n \leq -m$.*
(3) *$Ass_A M_n = Ass_A M_0$ for all $n \geq 0$.*

VIII (*Dimension of Supports and injective dimension:*) Let E be an A-module. Let $injdim_A E$ denotes the injective dimension of E. Also $Supp_A E = \{P \mid E_P \neq 0 \text{ and } P \text{ is a prime in } A\}$ is the support of an A-module E. By $\dim_A E$ we mean the dimension of $Supp_A E$ as a subspace of $Spec(A)$. We prove the following:

Theorem VIII *Let $M = H^i_I(R) = \bigoplus_{n\in\mathbb{Z}} M_n$. Then we have*

(1) *$injdim M_c \leq \dim M_c$ for all $c \in \mathbb{Z}$.*
(2) *$injdim M_n = injdim M_{-m}$ for all $n \leq -m$.*
(3) *$dim M_n = \dim M_{-m}$ for all $n \leq -m$.*
(4) *$injdim M_n = injdim M_0$ for all $n \geq 0$.*
(5) *$\dim M_n = \dim M_0$ for all $n \geq 0$.*
(6) *If $m \geq 2$ and $-m < r,s < 0$ then*
   (a) *$injdim M_r = injdim M_s$ and $\dim M_r = \dim M_s$.*
   (b) *$injdim M_r \leq \min\{injdim M_{-m}, injdim M_0\}$.*
   (c) *$\dim M_r \leq \min\{\dim M_{-m}, \dim M_0\}$.*

REGULARITY OF DETERMINANTAL THICKENINGS

CLAUDIU RAICU

We consider the ring $S = \mathbb{C}[x_{ij}]$ of polynomial functions on the vector space $\mathbb{C}^{m\times n}$ of complex $m \times n$ matrices. We let $\mathrm{GL} = \mathrm{GL}_m(\mathbb{C}) \times \mathrm{GL}_n(\mathbb{C})$ and consider its action via row and column operations on $\mathbb{C}^{m\times n}$ (and the induced action on S). The GL-invariant ideals $I \subseteq S$ have been classified and studied in the 80s, in work of De Concini, Eisenbud and Procesi [2]. In our work, for any such invariant ideal and for every $j \geq 0$ we compute the decomposition of the modules $\mathrm{Ext}^j_S(S/I,S)$ into irreducible GL-representations. Moreover, for any inclusion $I \supseteq J$ of GL-invariant ideals we determine the kernels and cokernels of the induced maps $\mathrm{Ext}^j_S(S/I,S) \longrightarrow \mathrm{Ext}^j_S(S/J,S)$, the study of which was motivated by work of Bhatt, Blickle, Lyubeznik, Singh and Zhang [1].

In my talk I plan to discuss two consequences of this work:

- I will explain how to determine the regularity of the powers and symbolic powers of generic determinantal ideals, and in particular I will characterize which powers have a linear minimal free resolution.
- I will characterize the GL-invariant ideals $I \subseteq S$ for which the induced maps $\mathrm{Ext}^j_S(S/I,S) \longrightarrow H^j_I(S)$ are injective, providing a partial answer to a question of Eisenbud, Mustaţă, and Stillman [3, Question 6.2].

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NORMAL HILBERT POLYNOMIALS

MARIA EVELINA ROSSI

In this talk we present a survey on the normal Hilbert coefficients of m-primary ideals of an analitically unramified Cohen–Macaulay ring A of dimension d. Most studies on this topic aim to extend the celebrated result by J. Lipman on a two-dimensional rational singularity. The papers by S. Itoh on the integral closures of ideals generated by regular sequences inspired recent results obtained jointly with A. Corso, K. Ozeki and C. Polini. In particular we give information on the first Hilbert coefficient which is strictly related to the geometric genus of A and we discuss interesting steps toward a conjecture by S. Itoh on the third Hilbert coefficient.

BALANCING IN RELATIVE CANONICAL RESOLUTIONS AND A UNIRATIONAL MODULI SPACE OF $K3$ SURFACES

FRANK-OLAF SCHREYER

I report on work of two of my students: Christian Bopp and Michael Hoff. Given a curve C of genus g together with a rational function $f : C \to P^1$ of degree d the canonical model lies on a rational normal scroll X, and the resolution of O_C as an O_X module is build with certain vector bundle N_i on P^1. It is interesting to ask whether the splitting type of the N_i is balanced for a general pair (C, f), since then jump loci lead to interesting subspaces of the Hurwitz scheme $H_{g,d}$. By experiment Bopp and Hoff discovered that the second syzygy bundle N_2 is not balanced for $(g,d) = (9,6)$ for finite fields. In the talk I will explain how their proof in characteristic zero builds upon a moduli space of certain lattice polarized $K3$ surfaces.

TEST IDEALS IN MIXED CHARACTERISTIC AND APPLICATIONS

KARL SCHWEDE

We will discuss some recent usage of Scholze's theory of perfectoid spaces in the study of singularities in mixed characteristic. One application that we will cover is uniform growth of symbolic powers of ideals in regular rings analogous to results of Ein–Lazarsfeld–Smith and Hochster–Huneke in characteristic zero and p respectively. This is joint work with Linquan Ma.

WEAK COMPLETE INTERSECTION IDEALS

JANET STRIULI

In this talk we will define a weak complete intersection ideal as an ideal with the property that every differential in their minimal free resolutions can be represented by a matrix whose entries are in the ideal itself. We extend several homological formulas that hold for the maximal ideal to weak intersection ideals.

LOCAL RINGS WITH QUASI-DECOMPOSABLE MAXIMAL IDEALS AND CLASSIFICATION OF THICK SUBCATEGORIES

RYO TAKAHASHI

A thick subcategory of a triangulated category is by definition a full triangulated subcategory closed under direct summands. Classifying thick subcategories of a triangulated category is one of the most important problems shared by homotopy theory [3, 5], ring theory [7, 10], algebraic geometry [9, 11] and representation theory [1, 4].

The singularity category (or stable derived category) of a noetherian ring R is defined as the Verdier quotient of the bounded derived category of finitely generated R-modules by perfect complexes. This triangulated category has been introduced by Buchweitz [2] to study maximal Cohen–Macaulay modules, and investigated by Orlov [8] in relation to homological mirror symmetry.

Let $(R, \mathfrak{m})$ be a commutative noetherian local ring. In this talk, we will first give a structure theorem of syzygies of modules over R when $\mathfrak{m}$ is decomposable. Applying this to the case where $\mathfrak{m}$ is *quasi-decomposable* (i.e. $\mathfrak{m}/(\underline{x})$ is decomposable for some regular sequence $\underline{x}$), we obtain several classfications of subcategories, including a complete classification of the thick subcategories of the singularity category of R. This talk is based on joint work with Saeed Nasseh [6].

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F-SIGNATURE OF CARTIER MODULES

KEVIN TUCKER

The F-signature of a local ring $(R, \mathfrak{m}, k)$ in positive characteristic $p > 0$ gives a measure of singularities by analyzing the asymptotic behavior of the number of splittings of large iterates of the Frobenius endomorphism. Assuming for simplicity that R is a domain of dimension d and $k = k^p$ is perfect, one decomposes

$$R^{1/p^e} = R^{\oplus a_e} \oplus M_e$$

as an R-module where $R \not| M_e$, yielding the maximal rank a_e of a free R-module quotient of R^{1/p^e} for each $e \in \mathbb{N}$. The F-signature $s(R)$, formally introduced by Huneke-Leuscke [HL02] after being studied implicitly by Smith-Van den Bergh [SVdB97], is then given by

$$s(R) = \lim_{e \to \infty} \frac{a_e}{p^{ed}}$$

and was shown to exist in full generality in [Tuc12]. Alternatively, as conjectured in [WY04] and shown in [PT16], one can view the F-signature as the (normalized) minimal relative Hilbert-Kunz multiplicity

$$s(R) = \inf_{I \subsetneq J} \frac{e_{HK}(I) - e_{HK}(J)}{\ell(J/I)}$$

over all collections of $\mathfrak{m}$-primary ideals $I \subsetneq J$. We highlight the following three important properties of the F-signature.

(1) $s(R) > 0$ if and only if R is strongly F-regular [AL03].
(2) $s(R) \leq 1$ with equality if and only if R is regular [HL02, Yao06].
(3) $\mathfrak{p} \mapsto s(R_\mathfrak{p})$ gives a lower semi-continuous function on $\operatorname{Spec}(R)$ [Pol15, PT16].

Attempts have also been made to define an analogous numerical invariant with respect to F-rationality. To that end, Hochster-Yao [HY] introduced the F-rational signature $s_{rat}(R)$ as the minimal relative Hilbert-Kunz multiplicity

$$s_{rat}(R) = \inf_{\substack{(I:x)=\mathfrak{m} \\ I \text{ parameter}}} (e_{HK}(I) - e_{HK}(I, x))$$

over all parameter ideals I and elements $x \in R$ representing an element of the socle of R/I. In a different direction, Sannai [San15] has defined the dual F-signature $s(\omega_R)$ using a variation on direct sum decompositions. Precisely, if b_e denotes the maximal number

$$(\omega_R)^{1/p^e} \twoheadrightarrow \omega_R^{\oplus b_e}$$

of direct sum copies of ω_R admitting a surjection from $(\omega_R)^{1/p^e}$, then one sets

$$s(\omega_R) = \limsup_{e \to \infty} \frac{b_e}{p^{ed}}.$$

Both $s_{rat}(R)$ and $s(\omega_R)$ satisfy the analogous property to (1) above: the are positive if and only if R is F-rational. However, it is unclear if $s_{rat}(R)$ satisfies (2) or (3). Similarly, while (2) holds for $s(\omega_R)$, it is unclear whether (3) holds – which stems from the lack of any control over the (possible) convergence of the sequence $\{\frac{b_e}{p^{ed}}\}$.

This talk will report on work in progress with Ilya Smirnov to propose a definition of the F-signature $s(\phi)$ of a Cartier module (M,ϕ), in the sense of [BB11]. A Cartier module (M,ϕ) is a finitely generated R-module M, together with a structure morphism $\phi\colon (M)^{1/p} \to M$. With mild assumptions on (M,ϕ) and R, this invariant can be used to detect F-regularity; moreover, suitably interpreted, it gives a lower semi-continuous function on $\mathrm{Spec}(R)$. The most important example of a Cartier module comes from considering the trace (or Grothendieck dual) of Frobenius $Tr_F\colon (\omega_R)^{1/p} \to \omega_R$. In this case, we refer to $s(Tr_F)$ as the Cartier signature, and one may view it as a (normalized) minimal relative Hilbert-Kunz multiplicity

$$s(Tr_F) = \inf_{\substack{I \subsetneq J \\ I \text{ parameter}}} \frac{e_{HK}(I) - e_{HK}(J)}{\ell(J/I)}$$

for any ideal J properly containing a parameter ideal I. As such, we observe the following string of inequalities

$$s_{rat}(R) \geq s(Tr_F) \geq s(\omega_R) \geq s(R).$$

Like the dual and F-rational signatures, the Cartier signature detects F-rationality (*i.e.* satisfies the analogue of (1)). Moreover, we are also able to show the Cartier signature satisfies properties (2) and (3) as well.

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THE GONALITY OF COMPLETE INTERSECTION CURVES

BROOKE ULLERY

Let C be a projective curve. Recall that the *gonality* of C, $\mathrm{gon}(C)$, is the minimum degree of a surjective morphism

$$\tilde{C} \longrightarrow \mathbb{P}^1,$$

where $\tilde{C}$ is the normalization of C. Thus, C is rational precisely when $\mathrm{gon}(C) = 1$, and, more generally, gonality measures how far the curve is from being rational. Gonality is a classical invariant, and there has been significant interest in bounding the gonality of various classes of curves and characterizing the corresponding maps to $\mathbb{P}^1$. Specifically, if C is embedded in projective space, it is natural to ask whether the gonality is related to the embedding of the curve.

In the case of complete intersection curves, the codimension two and three cases are thoroughly understood, due to theorems of Noether and Basili, respectively. Specifically, in these cases, the gonality is computed by projection from a linear space and every minimal covering arises in this way.

In my talk, I will discuss recent work with James Hotchkiss. Our main result is a generalization to higher codimension complete intersection curves. Specifically, we show that under mild degree hypotheses, the gonality of a general complete intersection curve $C \subset \mathbb{P}^n$ is computed by projection from an $(n-2)$-dimensional linear space, and any minimal degree branched covering of $\mathbb{P}^1$ arises in this way.